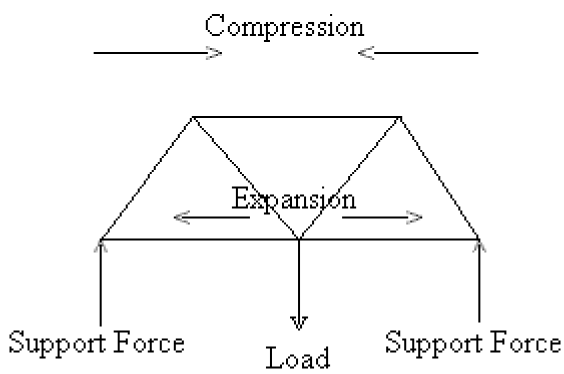
High School Mathematics-Physics SMILE Meeting
1997-2006 Academic Years
Mechanics: Bridges

09 December 1997: Professor John O'Leary [Civil and Architectural Engineering, IIT]

Professor O'Leary gave his annual lecture on principles of bridge-building.

He began by describing the forces on a typical bridge structure, with center loading and side supports:

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The effect of the load, and the balance of "bending moments" [what
Civil Engineers call "torques"] results in compression on the top
members, and forces of expansion at the bottom.
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The maximum stress [force per unit area] s_{max} on a board of width b, and height h, and length L is proportional to the length L, inversely proportional to the width b, and inversely proportional to the square of the height h:

$$s_{max}\ \mu\ L / b\, h^2$$

Furthermore, the bending displacement d at the center is proportional to the following:

$$d\ \mu\ L^3 / (b\, h^3)$$

As a consequence, we can make s_{max} and d small by making the height h large.

The buckling formula obtained by Euler is

$$P_{critical} = p^{2} E /4 I/L^{2}$$

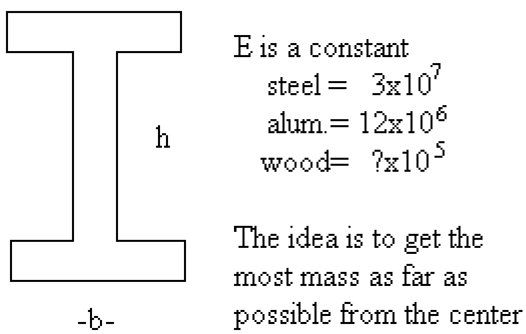
where the parameter I for a rectangle is

$$I = b h^{3}/12$$

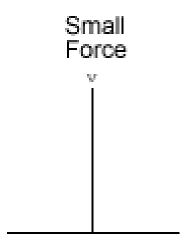
A beam buckles under compression about the "weak axis", rather than the "strong" axis.

Another important design principle is that "triangulation" makes very stable structures, whereas non-triangular regions can more easily become deformed under stress.

Comments by Alex Junievicz: J. O'Leary showed that the bending of a meter stick depended upon direction. Physics teachers call the bending torque, but civil engineers use the term Moments. The term torque has a different application to civil engineers. The problem in a bridge failure is usually twisting and x cross members are used in the base, top and entrance to the bridge. The top and bottom of an **I beam** are sometimes laminated to add thickness providing more strength.



He showed that a foam stick when presses bend and broke when pressure applied laterally

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Problem with tube bridges: twisting:
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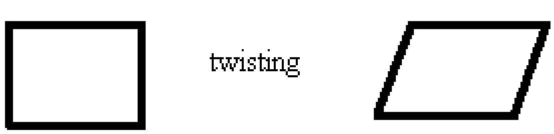
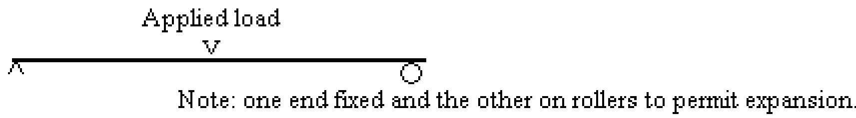
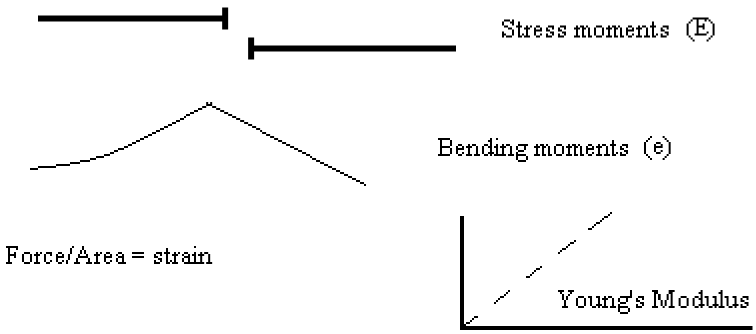


08 December 1998: Professor John O'Leary [IIT Department of Civil Engineering]

Bridge Design

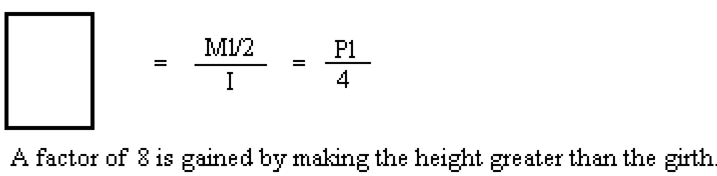
Discussion on bending and shear

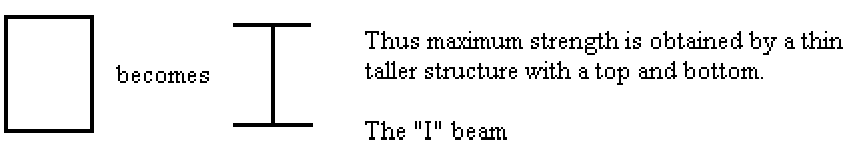


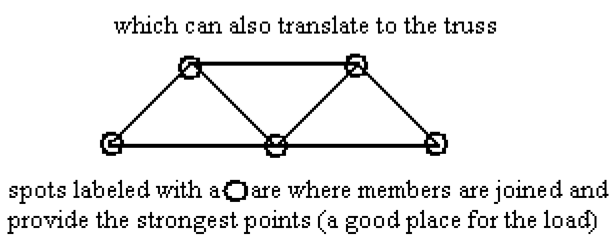


? = E e

-----------Geometry of Section-------------



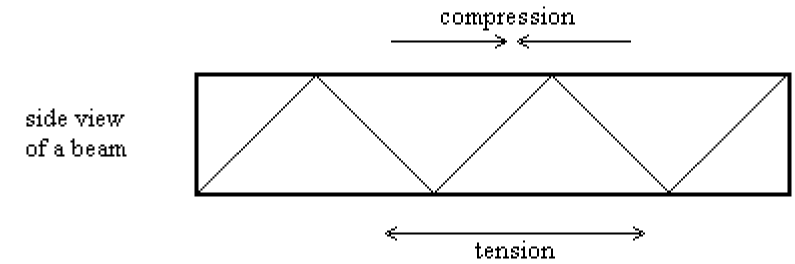




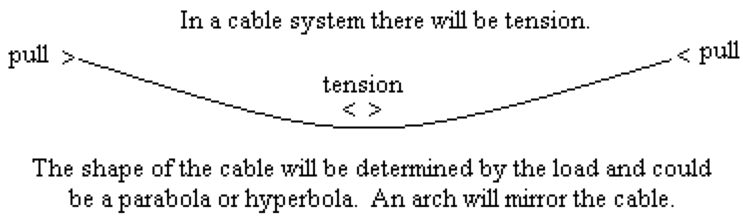
Note--Like a strip of paper it is strong in one plane and care must be made to prevent buckling (twisting) failures Note: bridges often fail by twisting, rather than from material failure.

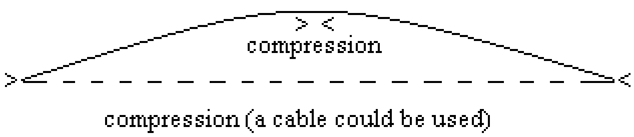
New Rules-abutments are possible. Now the bottom (or resting surface) can be fixed, as in the above one had to be variable, not it could be fixed and allow less materials used to prevent stretching when using an arch.

---------Compression vs Tension ------------



at the top there will be a force to compress; at the bottom there will be force to spread

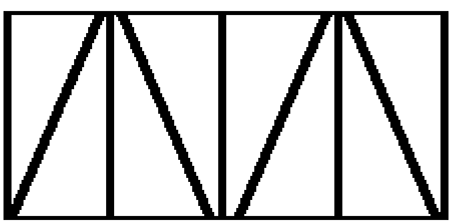




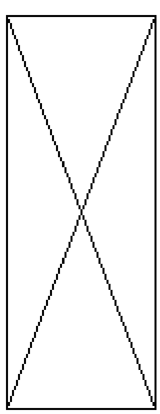
Interesting side Bar --after meeting

Load points should be junctions of members

| Load Points |
V V



side view



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x members help
prevent twisting failures
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References

- Why Buildings Stand up by Mario Salvodori
- Structures Daniel L Schodek 3rd Edition
- http://www.usma.edu [West Point Military Academy] has mechanical structures

Note: I saw an question someone had on a test where a locomotive 4 times the weight of a flat car hit and coupled, at 10Km/hr. My first thought was that there was a derailment and no conservation of movement on the tracks as they were dragged. I asked O'Leary's associate who seemed to be knowledgeable about Railroads....Coupling speed is done below 5 and at 10 damage may be result. I was interested in the comment that the power is much greater in toy trains that there may not be a realistic modeling.

07 December 1999: Eduardo De Santiago, Assistant Professor in IIT's Department of Civil and Architectural Engineering
introduced himself and explained that structural engineers design not only bridges, but are involved in design of nearly all structures: buildings, space platforms, dams, ships, antennas,... He soon had us involved in answering questions.

- What is a bridge?
- How would you define it?
- What is its function?

Dialogue established that a bridge provided a way for things (traffic, people, etc) to move across a chasm, river, etc. Consider a simple bridge: a plank supported at each end. If a load (person) stands at its center, what happens? A student provided an answer: It bends. And so it went. With a few sketches, Prof De Santiago soon had us considering bending moments (M) and shear forces (V) - both internal forces - and how they affect the plank. We defined internal stress from a sketch of the distribution of forces across a section of the plank, with the NA (neutral axis) on the geometric axis.

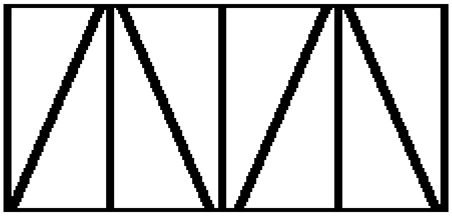
In less than an hour of give-and-take, he led us to develop an intuition for the factors that affect the strength of structures, and to the truss bridge. We came to understand these terms: **floor beam, panel, bottom and top chords, portal bracing, internal bracing**.

Three tips offered by Prof. De Santiago:

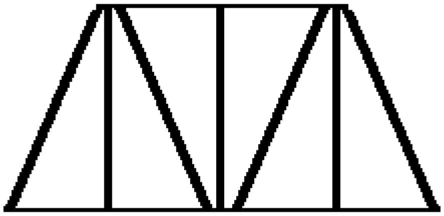
1. maintain symmetry
2. minimize the number of connections or joints
3. craftsmanship is very important

21 November 2000 Eduardo De Santiago [IIT Civil and architectural Engineering]
presented the **Bridge Design Lecture**, in preparation of the **Bridge Contest [http://www.iit.edu/~hsbridge/]** to be held at **IIT** on **13 February 2001**. He said that the simplest type of bridge is the "plank bridge" bridging a gap while supported on both ends. When you stop in the middle of the plank, it sags under your weight. The most noticeable effect is that of the **bending moment**, which causes the plank to "curl up". These bending moments are the most evident in bridge design, although shear forces [transverse action-reaction pairs at opposite ends of the board] are also important. The bending moment causes the top of the plank to be compressed and the bottom to be extended. The bending moment produces the greatest stresses at the top and bottom of the plank and decrease to zero at center. Therefore, the material in the center of the plank is being "wasted", since the greatest stress [force per unit area] is at the top [compression] and bottom [extension].

We may make a bridge more efficient by building a hollow beam with a few vertical supporting members [like a ladder turned on its side]. This construction reduces the effect of the bending moment, but increases that of shear stress. One may reduce the effect of shear forces by putting diagonal brace members into the network.



The object of design of wooden bridges is to convert bending moments and shear forces into longitudinal or axial forces [extension or compression], because wood is very strong under these forces. Also, we reduce the effect of shear by using triangles rather than rectangles, because rectangles collapse easily under shear, whereas triangles do not. Thus, the bridge geometry should consist entirely, or almost entirely, or triangles. In the above figure, you can remove the end members, which are subject to practically no stress, to simplify the construction to the following, known as a **truss**



A bridge consists of trusses on the sides, as well as a deck on the bottom. To avoid a collapse at the top, you should include bracing at the top, again consisting of triangles like the first figure. You should line up vertical joints exactly with horizontal joints to avoid "punch through". There should also be **lateral bracing** at the top, to avoid shear in the transverse structure.

The bridge should be "left-right symmetric", since is one side is weaker than the other it will break first. **Remember that the weakest part of the bridge always breaks first under loading.** These bridges are operating is a "near failure zone", which is not the regime in which large bridges are designed to operate. Engineers are necessarily conservative in their designs, and one should become an "anti-engineer" to win the contest.

These bridges may undergo "buckling", since wood is more resistant to tension than to compression. Under compression, a "slender" piece may buckle. Therefore, one should keep the compressional members short and fat.

Real bridges are also subject to "impact loads", produced by fast moving trains, trucks, winds, and even earthquakes. They can be ignored in the contest, so long as you remember "not to drop the weights on the platform", etc.

In the San Francisco Earthquake more than a decade ago, the lower deck of the Bay Bridge collapsed under action of a wave set up by the earthquake. The structural members of the bridge remained sound, however.

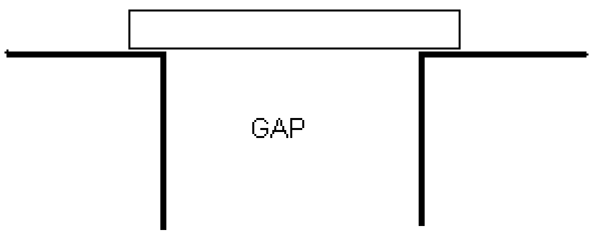
30 January 2001 Earl Zwicker (IIT)
indicated that **National Engineers Week** will occur during the period 18 - 24 February 2001. The IIT

Bridge Contest http://www.iit.edu/~hsbridge/ is officially connected with this celebration, and contest winners will be invited to a special banquet during that week. National Engineers Week, sponsored jointly by **IBM** and **NPSE**, has an official website, http://www.eweek.org/. The national organization is sponsoring the **Future City Competition**, as described in the website http://www.futurecity.org/.

20 November 2001: Eduardo de Santiago (Civil and Architectural Engineering, IIT)
Bridge Design Lecture for 2002 IIT Bridge Contest [http://www.iit.edu/~hsbridge/database/search.cgi/:/public/index]
Eduardo said that the goal of a **Structural Engineer** is to predict the forces acting on structures, and to determine whether those structures will collapse. He limited the discussion to **Truss Bridges**, addressing these basic questions:

- Why do they look the way they do?
- How do we make them stronger?

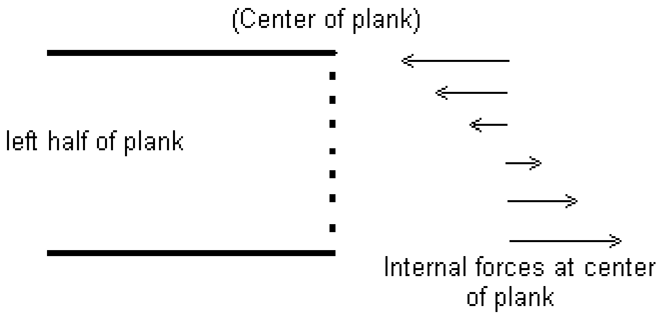
An old-fashioned bridge design might amount to putting a plank [or a tree] across a gap between two supports, as shown here:



This is not a very good bridge design, as can be seen in the "worst case" scenario by putting a significant load at the middle of the bridge. The bridge will bow in the middle if the load is substantial enough, because of the **Shearing Force** and the **Bending Moment**.

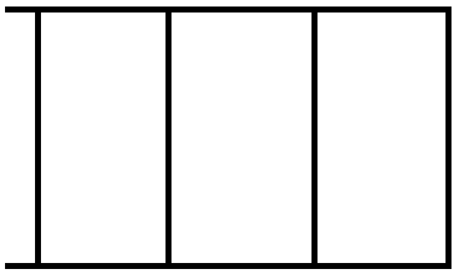
- The **shearing force** on a small segment acts "up" at one end and "down" at the other end, and tends to "slice through" the segment.
- The **bending moment** on a small segment acts clockwise on one end and counterclockwise on the other end, and tends to "bend" the segment.

The Bending Moment is most evident in practice; the plank bends as you walk across it. As viewed by a termite inside the middle of the plank the force changes gradually from compression to extension as you go through the plank from top to bottom, as shown:

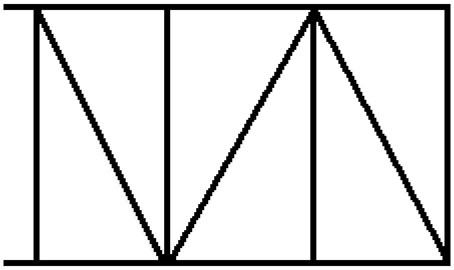


From an engineering viewpoint, the material along the edges of the plank is under the greatest distress [stress], so that it would constitute an improvement to "hollow out the beam":

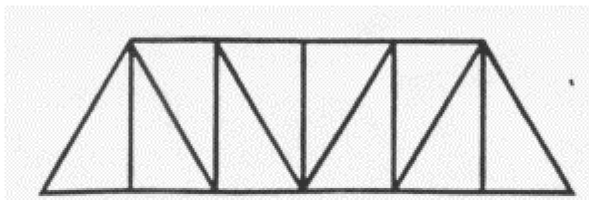
However, in such a case, the **top** part of the beam would carry **all the load**, and the **bottom** part would support **nothing**. Therefore, we insert vertical supports to transfer the load from the top to the bottom:



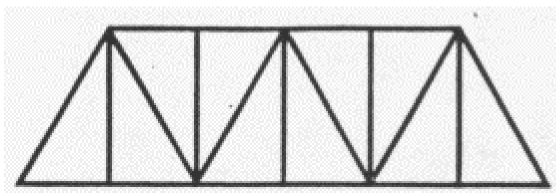
The **shear forces** would then cause a problem, and we must add diagonal members to transfer **both horizontal and vertical forces.** It is the **vertical components** that serve to reduce **shear forces**:



Craftsmanship is important in preparing these joints, in that it is important that the pieces fit together tightly, and that the joint members line up so that their centers meet at a point. The fundamental principle of Truss Design is to replace all **shear** and **bending** forces with **compression** and **extension** forces, and to reduce the structure to a **series of triangles.** There are several different types of basic bridge designs, such as these:



Pratt Design
Source: http://www.geocities.com/Baja/8205/truss.htm



Warren Design
Source: http://www.geocities.com/Baja/8205/truss.htm

Sunshine Skyway Cable Stay Bridge
Source: http://www.pbs.org/wgbh/nova/bridge/meetcable.html

A great deal of information is provided at the West Point Bicentennial Engineering Design Contest website, http://bridgecontest.usma.edu. In particular, you can design your bridge, and test it to find how and when it will fail. Also, you can download the following packet from that website:

Designing and Building File-Folder Bridges: A Problem-Based Introduction to

Engineering by Stephen J Rossler

This book provides students with an opportunity to learn how engineers use math, science, and technology to design real structures. It is intended primarily for high school students, but those in lower grades should be able to complete all but Learning Activity #3, which requires the application of geometry, algebra, and some basic trigonometry.

Eduardo mentioned that cross-bracing between trusses is required at their tops and bottoms. **Eduardo** gave the following tips and pointers:

- Make as few joints as possible.
- Be sure that there is a good fit at all joints.
- For crossed pieces, it is better to notch them slightly and glue them for a better fit, but don't make another joint there.
- Be sure to glue doubled sticks all along their lengths, and not just at the ends.

He closed with the following observations:

- Buttresses are good for bridges that permit support below the roadway, as is not often allowed in contests.
- Every bridge begins in the mind of an engineer.
- In earthquake engineering, **the idea is to save the people, even if the structure is severely damaged**. The idea is to make the building "ductile" (energy absorbing), and not necessarily "stiff". This way, the building can absorb energy without collapse, although it may be unusable after the earthquake. In a similar spirit, modern cars have "crumple zones" that are meant to crush and absorb energy, in contrast to old cars that remain intact in a collision but pass energy along to the occupants

04 December 2001: Ann Brandon (Joliet West HS, Physics)
Ann passed out a newspaper article describing an internet-based, virtual bridge building contest sponsored by the **U S Military Academy at West Point, NY**, using **West Point Bridge Designer** computer software available without cost at the contest website, http://bridgecontest.usma.edu. Students may compete either individually or in teams in this contest, which marks the bicentennial of the **USMA**. The prizes to winners are rather generous:

$15K [First], **$10K** [Second], and **$5K** [Third].

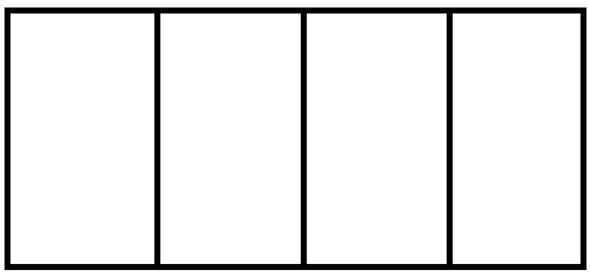
You may also obtain information by email: ic7097@usma.edu or by telephone at **1 - 845 - 938-2548**. Thanks, **Ann**!

19 November 2002: Professor Eduardo De Santiago [Civil and Architectural Engineering, IIT]
Bridge Design
Eduardo De Santiago [http://www.iit.edu/~santiago/] made his fourth annual presentation before **SMILE** and guest students and teachers on **"How to be a structural engineer in one lesson"!** He began by posing the following difficult question:

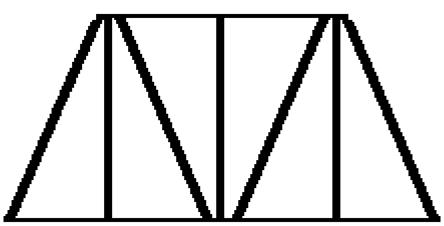
When and where will a given contest bridge fail?

He remarked that the answer to this question depends upon the details of the contest rules, craftsmanship in constructing the bridge, and other factors, although it seems that all good bridges up to now have been truss bridges. We will not repeat the discussion of why truss bridges are good, but refer to the relevant **SMILE** write-ups of **1999** [ph120799.htm], **2000** [mp112100.htm] and **2001** [mp112001.htm]. [See also the **Bridge Building Contest Home Page**: http://www.iit.edu/~hsbridge/database/search.cgi/:/public/index] Instead, we will simply list the relevant points that he made, in bullet form.

- To design a bridge for center loading, an optimal bridge will be symmetric about the center; that is, if your bridge is not symmetric, you are wasting material. In general, you should have a good understanding of the points at which the bridge may be loaded.
- When a bridge is supporting an external load, internal forces are developed in various parts of the bridge. Civil engineers analyze these forces in terms of **bending moments** and internal **shear forces**. A shear force tends to sever a beam, whereas a bending moment induces a deformation of the beam into a "smile" or a "frown". In general, bending moments are more significant than shears for bridge design.
- If you place a load on a horizontal plank placed between two abutments, the plank bows downward. In this situation the top part of the plank is under **compression,** and the bottom part of the plank is under **tension**. The "neutral axis" running horizontally along the center of the plank is under relatively weak internal forces. This idea is the basis for the "**I beam**" [transverse cross-section shaped like an **I**], in which the material is located primarily at the top and bottom of the beam, where the greatest internal stresses are found.
- Suppose we make a bridge that looks like a ladder turned on its side:



  This bridge will be resist bending, but will be very vulnerable to shear. We use trusses [diagonals] to handle shear forces efficiently.
- Here is a typical truss bridge panel



  The two good things about trusses are (1) that they can handle shear forces efficiently, and (2) that truss bridges --- assuming ideal "pin connections" --- are completely solvable, as well as generally strong structures. Note that the "triangulation" provides strength by preventing "buckling" of the bridge. [Since the ends of the bridge are supported by the abutment and do not experience a bending moment, a triangular portion at each end, being unnecessary, is removed.]
- A truss bridge is made by connecting two side panels, with cross-bracing and connections to provide triangulation at the top. In addition, portal bracing is required at the top to eliminate **side-sway**.
- As a material, wood is strong under tension, but has a strong tendency toward buckling under compression. Long, thin pieces of wood may be laminated by gluing them together along their entire length. This is especially important for the bottom members of the bridge.
- You should minimize the total number of joints, and be sure that the joints fit together snugly without

gaps, before gluing them. Remember that you are trying to obtain strength through triangulation. Alignment of joints is critical for building strong contest bridges.

- In practice, gusset plates may be used for strengthening joints in steel truss bridges, but these are probably not practical for contest bridges.
- It is better to have "butt joints" with the full member resting on top of the piece below, so that the wood, rather than merely the glue, is helping to support the weight of the bridge.
- You have to be sure that your weight platforms will support the weight by themselves, since there is the possibility of a "punch out", in which the bridge remains largely intact, while the platform punches through to release the weights.
- Some experimentation in the Seattle area has suggested that the best glue for contest bridges is ordinary wood glue [Elmer's Glue™?] , rather than the more expensive varieties.
- If weight must be supported below the roadbed, you can build an inverted truss. It is important to have bracing below ground level at the abutments. Remember that the Romans understood the arching effect, and also learned [sometimes the hard way] that you must have the arch well attached to prevent buckling. You can build an inverted arch, as well as the usual kind.
- Structural engineers are required to over-design bridges by safety factors, so that a 1000 kg load bridge will actually support 1500 kg, etc. Such caution is, or course, a sure way to lose bridge contests. The perfect contest bridge would resemble the legendary One Horse Shay [For details see **THE DEACON'S MASTERPIECE OR, THE WONDERFUL "ONE-HOSS SHAY": A LOGICAL STORY** by **Oliver Wendell Holmes** http://www.ibiblio.org/eldritch/owh/shay.html] That is, it would shatter to **smithereens** when it failed, since all its members would be equally pushed to the limit.
- Truss bridges are limited as to the distances they can span. For longer spans, either cable stay bridges or suspension bridges are required.
- Good luck to one and all on your bridge building!

A great deal of information is provided at the **West Point Bridge Design Contest** website, http://bridgecontest.usma.edu/index.htm. In particular, you can design your bridge, and test it to find how and when it will fail. Also, you can download the following packet from that website:
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02 December 2003: Professor Eduardo De Santiago [IIT: Civil and Architectural Engineering]
Building Lighter, Stronger Bridges
This was without doubt **Eduardo**'s most beautiful presentation yet! In preparation for the **28th Annual Chicago Regional Bridge Building Contests**, teachers and their students joined our **SMILE** meeting for **Eduardo's** fifth annual presentation on building a strong yet light bridge. **Eduardo's** previous lectures were given in **1999** [ph120799.htm], **2000** [mp112100.htm], **2001** [mp112001.htm],and **2002** [mp111902.html].---they dealt with the same ideas

He began by making a sketch on the board showing how a cave person would use a fallen tree as a primitive bridge to walk across a stream; a bridge takes a load from one point to another without breaking. Then he pointed out that a **truss** is a simple and efficient construction, relatively easy to analyze. He introduced the concepts of **bending moment, tension, and shear** as three important forces internal to a bridge, and he illustrated each concept with sketches and **by using whiteboard erasers that he bent and otherwise stressed**. From then on, one set of ideas led to another. **Eduardo** connected them with lucid sketches, eraser-bending, and articulate discussion. Simple ideas led to increasingly complicated combinations of ideas, and when we finally arrived at a typical **truss** bridge, we understood the physical ideas behind it. **He did not write down any equations!** He pointed out the need for **symmetry** (to deal with forces from any direction) and the need to **minimize the number of joints**. **And to have fun!**

When he had finished, he and some members of the **Bridge Building Committee** spent time with students and teachers **one-on-one, answering questions**. **What a wonderful, phenomenological presentation, Eduardo! Thanks!**

26 October 2004: George Krupa has indicated that there will be a **Future City Competition** in connection with **National Engineers Week**, in February 2005. For details see the website http://www.futurecity.org/. To see and/or download the handbook as a pdf file, go to http://www.futurecity.org/resources_handbook.shtm.

14 December 2004: Roy Coleman [Morgan Park HS, physics] Weighing Bridges for the Bridge Contest

Roy's students have been asking him how to determine whether their bridges weigh less than **28 grams**, as required under the rules for the **2005** Chicago regional bridge-building contests [http://www.iit.edu/~hsbridge]. He showed a simple balance set up with a meter stick balanced with its center on a cylindrical ball-point pen lying horizontally on the table. A few nickel coins served as "**precision weights**". The **mass** of each nickel is very close to **5 grams**. **Roy** took the bridge materials kit, placed it on one end of the meter stick balance, placed **6 nickels** on the other end of the stick, and found a good balance. He therefore concluded that the mass of the bridge materials was approximately **30 grams**. **Roy** showed that, by placing **5 nickels** at an end of the meter stick and one at **20 cm** from that end, one can determine whether the finished bridge weights less than **28 grams**. **Roy** mentioned that this was a good place to introduce a discussion of torques and their role in static equilibrium.

Nifty and nice, Roy!