REINFORCED CONCRETE ROOF
FOR THE
ROCKWELL ST. CAR HOUSE. C. C. R. CO.

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Design of a reinforced concrete roof for the
DESIGN OF A REINFORCED CONCRETE ROOF FOR THE ARCHER AVENUE AND ROCKWELL STREET CAR HOUSE OF THE CHICAGO CITY RAILWAY COMPANY.

A THESIS PRESENTED BY

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TO THE

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OF THE

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FOR THE DEGREE OF

CIVIL ENGINEER


[Signatures]

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The Archer avenue and Rockwell street Car House of the Chicago City Railway Company is located at the intersection of the streets which give it its name. By reference to the plan of the building shown on Plate I, it will be seen that the car house is a structure 490 ft. in length by 310 ft. in width. To reduce the risk of fire, the building is divided into eight (8) bays, separated by brick fire walls. One (1) bay known as the service bay, is devoted to the offices, boiler room, shop and wreck wagon room. The remainder of the building is used entirely for car storage.

In designing the roof of the car house, it was first decided to use steel trusses and a reinforced concrete slab roof. This method of construction was used at the 38th street & Cottage Grove avenue Car House. In order to make this construction fireproof, however, it was necessary to fireproof the trusses with metal lath and a prepared plaster. This construction was found to be more expensive and less satisfactory than reinforced concrete girders.

**THEORY OF THE REINFORCED CONCRETE TEE BEAM**

As shown on Plate VIII, the girders are both tee shaped and rectangular in section. On account of an 8 ft. continuous skylight in the center of each bay, the girders are rectangular in section between skylight curbs. In designing the girders, the formulae given in Turneaure & Maurer's "Principles of Reinforced Concrete Construction" were used. These formulae are based upon the tension in steel and are approximate. As stated by the authors, on page 80, there are no satisfactory formulae based on
the compression in the concrete.

The theory of the tee beam is as follows:

![Diagram of tee beam with notation]

**Notation**

- \( b \): width of flange
- \( d \): effective depth of beam
- \( b' \): width of web
- \( t \): thickness of flange
- \( c \): depth of neutral axis below top of flange
- \( x \): depth of resultant compression below top of flange
- \( f_s \): unit tension in steel
- \( f_c \): unit compression in concrete
- \( e_s \): unit elongation of steel due to \( f_s \)
- \( e_c \): unit shortening of concrete due to \( f_c \)
- \( E_s \): modulus of elasticity of steel
- \( E_c \): modulus of elasticity of concrete
- \( n \): ratio of \( E_s \) to \( E_c \)
- \( T \): total tension in steel
- \( C \): total compression in the concrete
- \( M_s \): resisting moment as determined by steel
- \( M_c \): resisting moment as determined by concrete
- \( k \): ratio of the depth of the neutral axis below the top to \( d \)
- \( j \): ratio of the arm of the resisting couple to \( d \)
- \( A \): area of cross section of steel
- \( p \): steel ratio \( A / d^2 \)

Since the modulus of elasticity is the ratio of the stress to the deformation we have

\[
-e_s = \frac{f_s}{E_s} \quad \text{and} \quad -e_c = \frac{f_c}{E_c}
\]

In order to derive a theory of flexure for concrete
beams, two \((2)\) assumptions are necessary, 1st Navier's hypothesis, that a plane section of a beam before bending is plane after bending; 2nd that the material obeys Hooke's Law that stress is proportional to strain.

The following deductions result from the above assumptions: First, that unit deformations at any section of a beam, are proportional to their distances from the neutral axis. Second, that the unit stresses are also proportional from the neutral axis.

From the above

With a comparatively thin slab, the neutral axis is generally below the bottom of the flange. It will be considered that all the compression in the concrete is carried by the flange. Unless the neutral axis is coincident with the bottom of the flange, this assumption is not true. However, the error is on the side of safety.

\[ \text{The average unit compression } y = \frac{f_c}{c} (c - \frac{1}{2} t) \]

This result is obtained by proportion in the triangle shown in Figure 2.

\[ y : f_c = (c - \frac{1}{2} t) : c \]
\[ cy = f_c (c - \frac{1}{2} t) \]
\[ y = \frac{f_c}{c} (c - \frac{1}{2} t) \]

The total compression = \(\frac{f_c}{c} (c - \frac{1}{2} t)bt\)
As the total compression equal the total tension

\[ f_c A = \frac{f_c}{c} \quad (c - \frac{t}{2}) \quad bt \]  

(2)

By eliminating \( \frac{f_c}{c} \) between equations (1) and (2) and solving for \( c \)

\[ c = \frac{2nd \ A + bt^2}{2(nA + bt)} \quad \text{where} \quad n = \frac{E_s}{E_c} \]  

(3)

By taking moments about the top of the flange, the value of \( x \), the distance to the center of gravity of the compressive area is found

\[ x = \frac{3c - 2t \cdot t}{2c - t} \cdot \frac{t}{b} \]  

(4)

With the resisting moment depending upon the steel,

\[ M_S = A f_s (d - x) \]  

(5)

\( (d-x) \) is the aim of the resisting couple.

From this equation \( f_s = \frac{M_S}{A (d-x)} \)  

(6)

With the resisting moment depending upon the concrete

\[ M_c = \frac{f_c}{c} \quad (c - \frac{t}{2}) \quad bt \quad (d - x) \].

From equation (1) substituting \( n \) for \( \frac{E_s}{E_c} \)

\[ f_c = \frac{f_s \cdot c}{n \cdot d - c} \]  

(7)

The application of these formulae will be considered under the subject of the design of reinforced concrete girders.

**THEORY OF RECTANGULAR REINFORCED CONCRETE BEAMS**

This theory is given in Turneanur & Maurer’s "Principles of Reinforced Concrete Construction". It is based on the linear
variation of the compression in the concrete. The notation is the same as that given for the tee beam.

As the unit deformations vary as their distances from the neutral axis,

\[ \frac{\varepsilon_s}{\varepsilon_c} = \frac{d-k\alpha}{kd} \]

**FIG 3**

\[ \varepsilon_s = \frac{f_s}{E_s} \]

and \( \varepsilon_c = \frac{f_c}{E_c} \)

(See the theory of the tee beam)

\[ \frac{f_s}{E_s} \frac{f_c}{E_c} = \frac{\varepsilon_s}{\varepsilon_c} = \frac{d-kd}{kd} = \frac{1-k}{k} \]

Substituting the value of \( n = \frac{E_s}{E_c} \)

\[ \frac{f_s}{n f_c} = \frac{1-k}{k} \]

The total tension on the section being equal to the total compression

\[ f_s A = \frac{1}{2} f_c \text{ bkd} \]

Eliminating \( \frac{f_s}{f_c} \) and substituting \( p \) for its value \( \frac{A}{bd} \), the following equations result,

\[ n \left( \frac{1-k}{k} \right) = \frac{k}{2p} \]

\[ 2pn (1-k) = k^2 \]

\[ k = \sqrt{2pn + (pn)^2 - pn} \]
This formula shows that the depth of the neutral axis depends entirely upon the percentage of steel and the ratio of the moduli of elasticities. The center of gravity of the compressive area, is at a distance of \( \frac{1}{3}kd \) from the top of the section. The lever arm of the resisting couple is equal to \( jd \).

\[
jd = d - \frac{1}{3}kd \quad j = 1 - \frac{1}{3}k \quad (11)
\]

If the resisting moment depends upon the steel,

\[
M_s = f_s A \times jd = f_s pjb^2 \quad (12)
\]

If the beam is over reinforced the compression in the concrete is the determining factor, and the resisting moment has the following value,

\[
M_c = \frac{1}{2}f_c bkd \times jd = \frac{1}{2}f_c kjbd^2 \quad (13)
\]

Solving equations (12) and (13) for \( bd^2 \),

\[
bd^2 = \frac{M}{f_s p j} \quad (14)
\]

\[
bd^2 = \frac{M}{f_c k j} \quad (15)
\]

**THEORY OF SHEAR AND BOND STRESSES**

This theoretical discussion is taken from the same authors as the preceding theories.

![Figure 4.](image)
\[ C' = C = \text{Total compression} \]
\[ T' = T = \text{Total tension} \]
\[ V = \text{Total vertical shear} \]
\[ v = \text{Unit horizontal shear} \]
\[ dL = \text{Short length of beam} \]
\[ jd = \text{Arm of resisting couple} \]
\[ b' = \text{Width of beam} \]

In this discussion the tension area in the concrete is neglected. The difference, between the tensile stresses \( T \) and \( T' \), will be equal to the total shear on any horizontal plane between the steel and the neutral axis. The unit shear would therefore be equal to \( \frac{T' - T}{B(dL)} \) or

\[ v = \frac{T' - T}{B(dL)} \]  (16)

Equating the moments of the shears,

\[ V (dL) = (T' - T) jd \]
\[ T' - T = \frac{V(dL)}{jd} \]  (17)

Substituting the value \((T' - T)\) in equation (16)

\[ v = \frac{V}{Bjd} \]  (18)

Taking the average value as \( .875 \) and substituting in (16)

\[ v = \frac{8}{7} \frac{V}{Bjd} \]  (19)

Formula (16) is applicable to tee beams as well as rectangular beams. If \((d - \frac{1}{2}t)\) is substituted for \(jd\), the error will be very small, and on the side of safety.

\[ v = \frac{V}{B(d - \frac{1}{2}t)} \]  (20)
As previously stated \( T' - T \) is the shear on any horizontal plane. Per unit of length, this shear will equal \( \frac{T' - T}{d} \).

If \( U \) equals the bond stress per lineal inch, then

\[
U = \frac{T' - T}{d} \quad \text{and} \quad \frac{V}{Jd}
\]

By dividing \( U \) by the sum of the perimeters of the steel sections, the bond stress per unit area of steel is obtained.

**SPACING OF GIRDERS**

A series of calculations each similar to those which follow, lead to the adoption of 16 ft. as the most economical spacing of the girders.

**BENDING MOMENT**

In order to determine the effect of settlement of the supports of a continuous slab roof an investigation was made of a roof slab of five (5) spans. The slab was assumed to have a thickness of 4-1/2". The weight of the roof was 55# per sq.ft.

The theory used in determining the bending moments of the continuous spans, was the graphic method developed by Prof. C. E. Greene in his "Trusses and Arches, Graphic Method Part II".

The four equations necessary for determining the pier ordinates for a beam of five (5) spans are as follows:

\[
H\left[ (2L+Lb)y + 2LbLz \right] = 6H\left( \frac{4a}{L} + \frac{Bb'}{Lb} \right) + 6EI\left( \frac{y_0 + y_e}{Lb} \right)
\]

\[
H\left[ ky + 2(Ld + 1c)y_2 + 1cy_2 \right] = 6H\left( \frac{Bb}{Ld} + \frac{Cc'}{Lc} \right) - 6EI\left( \frac{y_0 + y_e}{Ld} + \frac{y_0 + y_e}{Lc} \right)
\]  

(22)

(23)
\[ H[1cy_2 + 2(1c + 1d) y_3 + 1dy_4] = 6H(\frac{Cc}{1c} + \frac{Dd}{1d}) - 6EI(\frac{vC}{1c} + \frac{vD}{1d}) \]  
\[ H[1dy_3 + 2(1d + 1e)y_4] = 6H(\frac{Dd}{1d} + \frac{Eo}{1e}) - 6EI(\frac{vD}{1d} + \frac{vE}{1e}) \]

A reference to Plate II, and the following notation will explain the purpose of these equations:

- **H** = Pole distance in the stress diagrams, = 500#
- **l_a**, **l_b**, **l_c**, **l_d**, and **l_e** represent the lengths of the five (5) spans, in this case all equal to 16 ft.
- **y_1**, **y_2**, **y_3**, and **y_4** represent the ordinates under the supports in the equilibrium polygons. The purpose of the equations is the determination of the values of these ordinates. They are the only unknown quantities and when their values are found, the closing lines of the polygons may be drawn.

- **A**, **B**, **C**, **D**, **E** are the areas inclosed by the equilibrium polygons and the horizontal line **R**, **R_6**.

These areas are all equal to 40.5 sq.ft. in this problem.

- **a** and **a'** represent the distances from the center of gravity of the area **A** to the left and right-hand supports respectively.
- **bb', cc',** etc., represent similar distances for areas **B**, **C**, etc.

Where the supports are not on the same level, the deflections of the points of supports **v_a**, **v_b**, **v_c**, etc., below or above some line of reference must be taken into consideration.

In order to determine the effect on the adjacent spans, if one (1) support should settle, **R_3** was considered to have settled distances 1", 1/2" and 1/4"; all the other supports being on the same level. In considering equation (22) involving **l_a** and **l_b**, the
tangent or reference line passes through $R_2$. Therefore $v_b = 0$ and $v_c$ has a value of $+1''$, with $1''$ of settlement. For equation (23) the tangent passes through $R_3$. The values of $v_b$ and $v_d = -1''$. $E$ is the modulus of elasticity of the concrete, which was taken as 2,000,000. $I$ is moment of inertia of the section which was equal to 91.

Determination of $y_1$, $y_2$, $y_3$, and $y_4$ with the supports on the same level.

Substituting in equation (22)

$$64y_1 + 16y_2 = 6\left(\frac{324}{16} + \frac{324}{16}\right)$$

$$4y_1 + y_2 = 15.15$$

$$y_1 = \frac{15.15 - y_2}{4}$$

Equation (23)

$$y_1 + 4y_2 + y_3 = 6\left(\frac{324}{16} + \frac{324}{16}\right)$$

$$y_1 + 4y_2 + y_3 = 15.15$$

Equation (24)

$$y_2 + 4y_3 + y_4 = 15.15$$

Equation (25)

$$y_3 + 4y_4 = 15.15$$

Solving these equations, the following values were obtained:

$$y_1 = y_4 = 3.19 \text{ ft.}$$

$$y_2 = y_3 = 2.39 \text{ ft.}$$

Having the values of $y_1$, $y_2$, $y_3$, and $y_4$, it was only necessary to lay these distances off at the points of support as shown on
Plate II. The closing lines were drawn, connecting the extremities of the \( y \) ordinates.

1" settlement of \( R_3 \)

With the tangent passing through \( R_2 \), \( v_a = 0 \), \( v_c = +1" \)

Substituting in equation (22)

\[
500 \times 16(4y_1 + y_2) = 16 \times 15.15 \times 500 + \frac{6 \times 2,000,000 \times 91}{144} \times \frac{1}{16 \times 12}
\]

\[
4y_1 + y_2 = 15.15 + \frac{12,000,000 \times 91}{144 \times 16 \times 500} \times \frac{1}{16 \times 12}
\]

\[
4y_1 + y_2 = 20.08
\]

Equation (23) tangent through \( R_3 \)

\[
v_b = v_d = -1"
\]

\[
y_1 + 4y_2 + y_3 = 5.29
\]

Equation (24) tangent through \( R_4 \)

\[
v_c = +1" \quad v_e = 0
\]

\[
y_2 + 4y_3 + y_4 = 20.08
\]

Equation (25) tangent through \( R_5 \)

\[
v_d = v_f = 0
\]

\[
y_3 + 4y_4 = 15.15
\]

Solving the 4 equations for the 4 unknowns:

\[
y_1 = 5.31 \text{ ft.}
\]

\[
y_2 = -1.19 \text{ ft.}
\]

\[
y_3 = 4.65 \text{ ft.}
\]

\[
y_4 = 2.62 \text{ ft.}
\]

Having these values the closing lines were drawn, as in the previous case. For a settlement of \( 1/2" \) for \( R_3 \), the values
of the \( y \) ordinates are as follows:

\[
\begin{align*}
  y_1 &= 4.25 \text{ ft.} \\
  y_2 &= 0.6 \text{ ft.} \\
  y_3 &= 3.52 \text{ ft.} \\
  y_4 &= 2.91 \text{ ft.}
\end{align*}
\]

By comparing the values of \( y \), it is seen that a \( 1/2'' \) settlement of \( R_3 \) increases \( y \), from 3.19 ft. to 4.25 ft. or there is an increment of 1.06 ft. An additional settlement of \( 1/2'' \), increases \( y \), to 5.31 ft. on an increment of 1.06. This relationship is true for \( y_2, y_3 \) and \( y_4 \). Having established the values of \( y \) for any given settlement, it is possible to draw the closing lines for any settlement without further calculations. For each span, the closing lines have a common point of intersection as will be seen by referring to Plate II.

With the closing lines drawn, to obtain the bending moments, it is only necessary to scale the ordinates from the closing lines to the equilibrium polygons, multiplying these ordinates by \( H \). On Plate III is a table showing the maximum positive and negative bending moments, and also the points of inflection for various conditions of the supports. From the table it is seen that for the intermediate panels the maximum positive moment is \( \frac{wL^2}{241.7} \) and the maximum negative moment \( \frac{wL^2}{12.9} \).

In designing the roof slab, on account of the possible lack of full continuity, a compromised value for the positive bending moment was used. This value was midway between \( \frac{wL^2}{8} \) for a simple span and \( \frac{wL^2}{241.7} \) for a continuous span, or \( \frac{wL^2}{14.4} \).
For the negative moment a value of $\frac{wL^2}{10}$ was used. This value would take care of a possible settlement of 1/4".

**DESIGN OF THE REINFORCED CONCRETE ROOF SLAB**

The thickness of the slab was taken as 4-1/2", giving a dead load of 55# per sq. ft. With a snow load of 25# per sq. ft., the total load becomes 80# per sq. ft.

Reinforcement to resist positive bending moment. Width of slab 12".

$$M = \frac{wL^2}{14.4} = \frac{80 \times 256 \times 12}{14.4} = 17,100 \text{ in. #}$$

Formula (12) gives the required area of steel

$$A = \frac{M}{f_S j d}$$

$j$ depends upon $k$, $p$ and $n$. The value of $j$ can be calculated from the formulae (10) and (11) previously given. It is much simpler to use the diagram given on Plate V which was taken from Turneaure & Maurer's "Principles of Reinforced Concrete Construction" page 57.

Assume $p = .0083$ and $n = 15$. From the diagram for $p = .0083$ and $n = 15$, $j = .875$ and $k = .390$.

$$d \text{ the effective depth is 3.5" } f = 16,000\# \text{ per sq. in.}$$

$$A = \frac{17,100}{1600 \times .875 \times 3.5} = .35 \text{ sq. in.}$$

Considering only the concrete above the center of the reinforcement, the area per ft. width equals 12x3.5 = 42 sq. in.

$$p = \frac{.35}{42} = .00833$$
The value assumed of $\mu$ is correct. This percentage can only be obtained by trial. The spacing for 1/2" square rods is as follows: The area of a 1/2" square rod is .25 sq. in. The number of rods required per ft. = \( \frac{35}{25} = 1.4 \) \( \frac{12}{1.4} = 8\text{-1/2" spacing} \).

8" spacing was actually used in the roof.

Reinforcement to resist negative bending moment.

Assume $p = .012$ \( j \) therefore equals .85 and $k$ equals .445

\[
M = \frac{w l^2}{10} = \frac{80 \times 256 \times 12}{10} = 24,600 \text{ in. \#}
\]

\[
A = \frac{24600}{16000 \times 85 \times 3.5} = .516 \text{ sq. in.}
\]

\[
p = \frac{.516}{42} = .0123
\]

Spacing of 1/2" square rods

\[
\frac{.516}{.25} = 2.06 \text{ rods per ft.} \quad \frac{12}{2.06} = 5.83"
\]

6" spacing was used on the roof.

**BOND**

The stress on the bottom rods resisting the positive bending moment, becomes zero at the points of inflection. Taking the shear at the point which averages 4 ft. from the supports the following bond stresses result:

\[
U = \frac{V}{jd}
\]

\[
V = 80 \times 4 = 320\# \quad j = .875 \quad d = 3.5
\]
\[ U = \frac{320}{0.875 \times 3.5} = 104.5\# \]

For 8" spacing the sum of the perimeters per ft. width of slab
\[ = 2 \times 1.5 = 3 \text{ inches.} \]

\[ \frac{104.5}{3} = 34.8\# \text{ per sq. inch} \]

For the top or negative reinforcing rods
\[ U = \frac{640}{0.85 \times 3.5} = 215\# \]

For 6" spacing the sum of the perimeters \[ = 2 \times 2 = 4 \text{ inches.} \]

\[ \frac{215\#}{4} = 53.75\# \text{ per sq. inch} \]

These values for bond stresses are not high. For Johnson
corrugated bars, which were used in the roof, the experiments of
the University of Illinois given in their Bulletin #8, show that
the average ultimate bond stress to be 595\# per square inch for
1/2" square bars.

**SHEARING STRESSES**

From formula (19)

\[ v = \frac{8V}{7b d} \]

The maximum shear \[ V = 640\# \]

\[ v = \frac{8 \times 640}{7 \times 12 \times 3.5} = 17.4\# \text{ sq. inches} \]

**END PANEL**

The calculations for the end panels are exactly similar
to those for the intermediate spans, except for the value of the
bending moment. The value of M was taken as \[ \frac{wL^2}{8} \]. The length
of the end panels is 13 ft. and the spacing of the bars 5-1/2".

TEMPERATURE AND SHRINKAGE STRESSES

The amount of reinforcement necessary to resist the temperature and shrinkage stresses can only be roughly estimated. With a temperature range of 50° F. the stress in the steel would be 50x.0000065x$E = 9750$# per sq. inch, where the coefficient of expansion for steel is .0000065 per degree F. Large cracks cannot form in the concrete, unless the steel is stretched beyond its elastic limit. Assume the concrete to have a tensile strength of 200# per sq. inch, and the elastic limit of the high carbon steel is 60000# per sq. inch

$$p = \frac{200}{60000-9750} = .004$$

To resist temperature and shrinkage stresses transverse to the direction of the main reinforcement, 1/2" square corrugated bars were arbitrarily spaced 1 ft. apart. This corresponds to a value of $p$ of .006. In the direction of the span, additional reinforcement was added by running every third one of the top rods the entire length of span, and every second one of the bottom rods, as shown on Plate VIII.

SKYLIGHT CURB

Loads:

Assume the curb to carry 2 ft. of the adjacent slab:

- Skylight 125# per ft.
- Snow 50# per ft.
- Slab 160# per ft.

335# per ft.
\[ M = \frac{wl^2}{8} = \frac{335 \times 2.56 \times 12}{8} = 129000 \text{ inch lbs.} \]

With a width of curb of 8"; and with \( p = 0.0125 \)
\[ d^2 = \frac{129000}{8 \times 3.5 \times 16000 \times 0.0125} = 95 \]

where \( k = 0.45 \) and \( j = 0.86 \), \( d = 9.75" \)

Total depth 11.75"

Reinforcement of two (2) - 7/8" round rods.

**DESIGN OF THE REINFORCED CONCRETE GIRDERS**

With a girder spacing of 16 ft., the roof load to be carried amounts to 16x80# or 1280# per ft. or girder. The dead load of the girder must first be assumed. Having made several trial calculations for the required depth with a given width, the dead load can be determined exactly, and the depth refigured.

Assume the width of the beam to be one (1) ft. Owing to the pitch of the roof, which is 1" per ft. the dead load varies from a minimum at the support to a maximum in the center.

As each bay of the car house has an eight (8) ft. continuous skylight, the tee shaped girder is only effective from the points of support up to the skylight. On Plate IV the dead and live loads of the girder are shown.

For the Tee section, a certain width of slab may be considered as a part of the girder. Just how much of the slab is effective, it is impossible to determine. Various formulae and rules have been proposed for determining the breadth of slab. The Prussian Regulations of 1904, give the permissible width as less than one-third (1/3) the length of the span; the French and British rules, less than three-quarter (3/4) spacing...
of the beams. If these rules are applied to the case under consideration, it would mean that anything under 12 ft. could be considered as the effective width of the slab. In the June, 1908 Bulletin of the Corrugated Bar Company, a table of formulae for tee beams is given. For 4-1/2" slab, with an effective girder depth of 24", the formula is \( b = 0.152 \left( l - \frac{1}{2} t \right) \). \( b' \) = breadth of beam = 12". \( l \) = length of span = 37 ft. \( b = 62.5" \). The value of \( b \) used in designing the girders was 4 ft.

From equation (3) \( C = \frac{2nd A + bt^2}{2(nA + bt)} \)

and \( x = \frac{3c - 2t}{2c - t} \frac{t}{3} \) (4)

In order to facilitate the solution of these formulae, a diagram was made (Plate XV), which gives values of \( C \) and \( x \) for various values of \( A \) and \( d \). The quantities \( n \), \( b \) and \( t \) are constant for this particular problem. \( n = 15 \) \( b = 48" \) and \( 4.5" \). The intersection of the lines representing the depth of the girder and the area of the steel, is horizontally projected to the right giving the proper value of \( C \). This value of \( C \) when projected horizontally to an intersection with the curve on the right-hand side of the diagram, will give the proper value of \( X \) vertically above the point of intersection.

The bending moments for designing the various sections of the girder are found from the diagram on Plate XVI. Sections were taken two (2) ft. apart corresponding with the points at which the loads were assumed to be applied. Section #1 Ordinate = 82" \( H = 40000 \). Bending moment = \( 82 \times 40000 = 3,280,000 \) inch.
pounds.

\[ bd^2 = \frac{M}{f_s \cdot \frac{p}{j}} \quad \text{Formula (12)} \]

\[ bd^2 = \frac{M}{\frac{1}{4} f_c \cdot \frac{k}{j}} \quad \text{Formula (13)} \]

Applying (12)

\[ d^2 = \frac{3,280,000}{16000 \times 0.0115 \times 0.35 \times 12} = 1748 \]

\[ d = 41.8" \]

Applying (13)

\[ d^2 = \frac{3,280,000}{350 \times 44 \times 0.35 \times 12} = 2083 \]

\[ d = 45.5" \]

From formula (12) the depth is dependent upon the steel and in (13) the compression in the concrete is the controlling factor. The maximum value obtained was 45.5 inches for the effective depth. In the design the effective depth was taken as 45" and the total depth 48". In solving the above formulae, the following values were used \( f_s = 16000 \# \), \( f_c = 700 \# \), \( p = 0.0115 \), \( j = 0.85 \), \( k = 0.44 \). The value of \( p \) was found after several trials. It corresponds to a reinforcement of 6.12 sq. inches or eight (8) - 7/8" square rods. Having assumed \( p \), the corresponding values of \( j \) and \( k \) were found from Plate V, where \( n = 15 \). Having assumed eight (8) 7/8" square rods for the reinforcement, the stresses in the steel and concrete will be calculated.
Section #3. The depth at this point is the result of several trials. Having determined upon the effective depth of 36.5", the depths of all sections were fixed by the slope of 1" to the foot of the roof. Referring to Plate V, \( C = 12.5" \) and \( X = 2.08" \) for \( d = 36.5" \) and \( A = 6.2 \text{ sq.in.} \). Substituting these values in (6) and (7)

\[
fs = \frac{MS}{A(d-X)} = \frac{3,160,000}{6.2 \times 34.42} = 14800\#
\]

\[
f_c = \frac{f_s \cdot c}{n(d-c)} = \frac{14800 \times 12.5}{15 \times 24} = 514\#
\]

Section #4

\[
d = 34.5\" \quad C = 11.8\" \quad X = 2.07\" \quad A = 6.2 \text{ sq.in.}
\]

\[
fs = \frac{3,080,000}{6.2 \times 32.45} = 15300\#
\]

\[
f_c = \frac{15300 \times 11.8}{15 \times 22.7} = 530\#
\]

Section #5

\[
d = 32.5\" \quad C = 11.5\" \quad X = 2.065\" \quad A = 6.2 \text{ sq.in.}
\]

\[
fs = \frac{2840,000}{6.2 \times 30.435} = 15000\#
\]

\[
f_c = \frac{15000 \times 11.5}{15 \times 21} = 548\#
\]

Section #6

\[
d = 30.5\" \quad C = 10.7\" \quad X = 2.05\" \quad A = 6.2 \text{ sq.in.}
\]

\[
fs = \frac{2,520,000}{6.2 \times 28.45} = 14300\#
\]

\[
f_c = \frac{14300 \times 10.7}{15 \times 19.8} = 515\#
\]

Section #7

\[
d = 28.5\" \quad C = 10.2\" \quad X = 2.03\" \quad A = 6.2 \text{ sq.in.}
\]

\[
fs = \frac{2,080,000}{6.2 \times 25.47} = 12700\#
\]

\[
f_c = \frac{12700 \times 10.2}{15 \times 18.3} = 472\#
\]
The stresses being low at this section with eight (8) 7/8" square bars, two (2) bars were turned up at section #7a, 1 ft. beyond section #7. The stresses for section #7a with six (6) bars are as follows:

\[
d = 27.5" \quad C = 8.4" \quad X = 1.97" \quad A = 4.6 \text{ sq. in.}
\]
\[
f_s = \frac{1,750,000}{4.6 \times 28.53} = 14900# \\
f_c = \frac{14900 \times 6.4}{15 \times 19.1} = 438# \\
\]

Section #8
\[
d = 26.5" \quad C = 8.2" \quad X = 1.97" \quad A = 4.6 \text{ sq. in.}
\]
\[
f_s = \frac{1,560,000}{4.6 \times 24.53} = 13320# \\
f_c = \frac{13320 \times 8.2}{15 \times 18.3} = 413# \\
\]

Section #9
\[
d = 24.5" \quad C = 6.2" \quad X = 1.83" \quad A = 3.1 \text{ sq. in.}
\]
\[
f_s = \frac{1,000,000}{3.1 \times 22.67} = 14200# \\
f_c = \frac{14200 \times 6.2}{15 \times 18.3} = 321# \\
\]

The above stresses indicate that it is safe to turn up the second pair of bars at this point, leaving four (4) bars to run horizontally the entire length of the girder.

Section #10
\[
d = 22.5" \quad C = 6.0" \quad X = 1.81" \quad A = 3.1 \text{ sq. in.}
\]
\[
f_s = \frac{240,000}{3.1 \times 20.69} = 3740# \\
f_c = \frac{3740 \times 6}{15 \times 16.5} = 90# \\
\]
SHEARING STRESSES

The shearing stresses are tabulated on Plate VII. The total shear $V$ was taken from the shear diagram on Plate IV. Equation (20) gives the unit shear for a tee beam $v = \frac{V}{b'(d+2t)}$. As an example of calculating the area required for the stirrups, section #8 will be considered. The method of determining the number and size of the stirrups is only an approximation and not exact. It is generally considered that a unit shearing stress of 30# per square inch in the concrete is safe. When the stresses are greater than this, it is necessary to provide steel to take care of the shear. In making the calculations for the girder, steel was provided to take care of the unit shearing stresses above 30# per square inch. In section #8, $v = \frac{22600}{291} = 77.60#$ per sq. in. $77.60# - 30# = 47.60#$

The amount of steel required in a length equal to d or $26.5" = (v-30)b'(d+2t) = \frac{47.60 \times 291}{16000} = .865$ sq. in.

The stirrups used in the girder were made up of two (2) U shaped sections of 1/2" round corrugated bars. Each stirrup had a total cross sectional area of .7854 sq. in. If .865 sq. inches of steel were required in length of 26.5 in., the theoretical spacing of the stirrups would be 24 inches. In the design, the theoretical spacing was not adhered to, as the stirrups were placed much closer than required by the calculations. The reason for this, was the uncertainty of any determinations of this kind. Furthermore vertical stirrups were used, which are not as effective as inclined rods. Vertical stirrups, however, are easier
to place in position and interfere less with the filling of the forms with concrete. No account was taken of inclined reinforcing bars in resisting shear.

FORMS

Details of the forms are shown on Plate VIII. Long Leaf Yellow Pine was used for the forms except for the 1" boards and some of the small braces. For roof boards, Hemlock was used and Norway Pine for the braces. To facilitate handling the forms, girder bottoms were made in three (3) sections; and the sides in five (5) sections. The 6"x6" posts, two (2) for each girder, carried a load of 16,500# each. The actual cross section of the posts was 30.25 sq. in., giving a unit compression of 545# per sq. in. This load was distributed over a plank foundation having an area of 16 sq. ft. The average settlement of the posts when the forms were filled was 3/8 of an inch. To take care of this settlement the girders were given a camber of 1". When the roof was completed, it was found that all the heavy form lumber of 2" thickness or more was practically as good as new, after having been used three (3) times. The lighter material had no value after being used the same number of times.

CONSTRUCTION

The concrete for the roof was a mixture of 1 : 2-1/2 : 5, the stone being small enough to pass through a 1/2" ring. Two (2) machines were used for mixing the concrete, which was hoisted to the roof in wheel-barrows. The roof slab was made of 4" of concrete on top of which was placed 1/2" of 1 : 3 cement mortar.
The mortar was used in order to make the roof watertight, as the roof was not covered with any waterproof material. Joints between one day's work and the next were placed half-way between girders where the shear was zero. For the concrete that was placed in good weather, the slab forms and girder sides were left in place for ten (10) days, and the girder bottoms and posts for three (3) weeks. For work done in freezing weather the slab forms were left in place for four (4) weeks and the girder posts for six (6) weeks. Photographs taken by the writer, when acting as Engineer in Charge of construction, are shown on Plates IX, X and XI.

**EXPANSION JOINTS**

Provisions for expansion were made at all points of bearing. Strips of #18 gauge galvanized iron were placed over all walls to break the bond between the concrete slab and the brick work. On Plate VIII, details are shown of the expansion joints at the end walls. With a temperature range of 50 degrees F. the theoretical expansion and contraction would be nearly 2 inches for a roof 490 ft. long.

\[
\text{Expansion per degree } F. = 0.000006 \\
\text{Expansion for 50 deg. } F. = 0.0003 \\
0.0003 \times 490 = .147 \text{ ft.} = 1.76" 
\]

Sufficient space was allowed over the walls for an expansion of two (2) inches. The actual movement of the slab between summer and winter averaged 3/4 of an inch. As the car house is heated in winter, the temperature of the roof slab does not vary 50° F. during the year.
ROCKWELL STREET.

Capacity Inside 210 D.T. Cars.
Capacity Outside 28 D.T. Cars.
Total Capacity 238 D.T. Cars.

PLATE I
PLAN OF THE ARCHER AVE. & ROCKWELL ST.
CAR HOUSE, CHICAGO CITY RAILWAY CO.
Scale: 1" = 50 ft.
Thesis of C. D. Williams, 1910.
### EFFECT OF SETTLEMENT of ONE SUPPORT on A CONTINUOUS BEAM OF FIVE SPANS

**Length of all spans:** 16'-0". **Width of beam:** 1'-0". **Uniform load:** 55' per sq. ft. = W. + = positive bending moment. - = negative. C1, C2 etc. = point of max. + bending moment.

#### Table: Bending Moments and Reactions

<table>
<thead>
<tr>
<th>Point</th>
<th>Bending Moments Coefficients of WL²</th>
<th>Reactions Coefficients of WL</th>
<th>Ratio of Distance of Pt. of Inflection to Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal Settlement of R₃</td>
<td>Normal Settlement of R₃</td>
<td>Normal Settlement of R₃</td>
</tr>
<tr>
<td>1</td>
<td>0 0 0 0 0</td>
<td>.396  .380  .365  .334</td>
<td>3.38 ft.  3.90 ft.  4.50 ft.  5.60 ft.</td>
</tr>
<tr>
<td>2</td>
<td>1 12.8 + 1 14</td>
<td>.396  .380  .365  .334</td>
<td>3.38 ft.  3.90 ft.  4.50 ft.  5.60 ft.</td>
</tr>
<tr>
<td>3</td>
<td>2 13.7 - 2 35</td>
<td>1.035  1.109  1.260  1.380</td>
<td>4.25 ft.  4.50 ft.  4.75 ft.  5.10 ft.</td>
</tr>
<tr>
<td>4</td>
<td>3 30.7 + 3 24.7</td>
<td>1.352  1.409  1.538  1.630</td>
<td>3.45 ft.  2.25 ft.  1.10 ft.  -</td>
</tr>
<tr>
<td>5</td>
<td>4 21.7 + 4 21.2</td>
<td>1.352  1.409  1.538  1.630</td>
<td>3.20 ft.  2.05 ft.  1.00 ft.  -</td>
</tr>
<tr>
<td>6</td>
<td>5 12.9 - 5 28.0</td>
<td>1.352  1.409  1.538  1.630</td>
<td>3.20 ft.  3.60 ft.  3.95 ft.  4.50 ft.</td>
</tr>
<tr>
<td>7</td>
<td>6 21.7 + 6 21.2</td>
<td>1.352  1.409  1.538  1.630</td>
<td>3.45 ft.  4.33 ft.  5.25 ft.  7.75 ft.</td>
</tr>
<tr>
<td>8</td>
<td>7 13.7 - 7 35</td>
<td>1.352  1.409  1.538  1.630</td>
<td>4.25 ft.  4.50 ft.  4.65 ft.  5.25 ft.</td>
</tr>
<tr>
<td>9</td>
<td>8 30.7 + 8 24.7</td>
<td>1.352  1.409  1.538  1.630</td>
<td>3.45 ft.  4.33 ft.  5.25 ft.  7.75 ft.</td>
</tr>
<tr>
<td>10</td>
<td>9 30.7 + 9 35</td>
<td>1.352  1.409  1.538  1.630</td>
<td>3.45 ft.  4.33 ft.  5.25 ft.  7.75 ft.</td>
</tr>
<tr>
<td></td>
<td>10 12.9 - 10 28.0</td>
<td>1.352  1.409  1.538  1.630</td>
<td>3.45 ft.  4.33 ft.  5.25 ft.  7.75 ft.</td>
</tr>
</tbody>
</table>

#### Notes
- Plate III
- Thesis of J. C. Willard
PLATE V
VALUES OF K AND U
FROM TURNEAURE & MAURER'S
"PRINCIPLES OF REINFORCED CONCRETE
CONSTRUCTION"
THESIS OF C. MILLARD
MAR. 7, 1910.
NOTE: THIS DIAGRAM IS ONLY APPLICABLE TO A TIE BEAM WHERE THE CONSTANTS HAVE THE FOLLOWING VALUES: \( a = 15 \), \( b = 45^\circ \), \( c = 45^\circ \).

DIAGRAM FOR SOLUTION OF EQUATIONS

C. STODDARD 30

PLATE VI

THESIS OF C. STODDARD, MAR 1916
## Shearing Stresses

<table>
<thead>
<tr>
<th>Section</th>
<th>Effective Depth $d$</th>
<th>Total Shear $V$</th>
<th>$l'(d-\ell t)$</th>
<th>Unit Shear $\gamma = \frac{V}{l'(d-\ell t)}$</th>
<th>Area of Steel $A$, Required for Shear in Length $d$</th>
<th>Theoretical Spacing of Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45.0&quot;</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>45.0&quot;</td>
<td>1200#</td>
<td>0.0</td>
<td>2.5 # sq.in.</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>36.5&quot;</td>
<td>6200#</td>
<td>411</td>
<td>15.10#</td>
<td>0.131 # sq.in.</td>
<td>194&quot;</td>
</tr>
<tr>
<td>4</td>
<td>34.5&quot;</td>
<td>9500#</td>
<td>387</td>
<td>24.60#</td>
<td>0.378&quot;</td>
<td>63&quot;</td>
</tr>
<tr>
<td>5</td>
<td>32.5&quot;</td>
<td>13000#</td>
<td>363</td>
<td>35.80#</td>
<td>0.623&quot;</td>
<td>36&quot;</td>
</tr>
<tr>
<td>6</td>
<td>30.5&quot;</td>
<td>16200#</td>
<td>339</td>
<td>47.80#</td>
<td>0.865&quot;</td>
<td>24&quot;</td>
</tr>
<tr>
<td>7</td>
<td>28.5&quot;</td>
<td>19400#</td>
<td>315</td>
<td>61.60#</td>
<td>1.085&quot;</td>
<td>17&quot;</td>
</tr>
<tr>
<td>8</td>
<td>26.5&quot;</td>
<td>22600#</td>
<td>291</td>
<td>77.60#</td>
<td>1.400&quot;</td>
<td>12&quot;</td>
</tr>
<tr>
<td>9</td>
<td>24.5&quot;</td>
<td>25400#</td>
<td>267</td>
<td>95.10#</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AT SUPPORT</td>
<td>21.5&quot;</td>
<td>29350#</td>
<td>231</td>
<td>127.00#</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Sections 1 and 2 are rectangular, shear calculated from formula $\gamma = \frac{V}{ld}$
S.E. CORNER OF THE ARCHER & ROCKWELL CAR HOUSE.

INTERIOR OF A BAY
GIRDER FORMS

ROOF FORMS FROM BELOW

PLATE X
PLATE XI

GIRDER & ROOF SLAB REINFORCEMENT

JOISTS FOR ROOF SLAB FORMS